

## HYBRID DYNAMIC-STATIC FINITE-DIFFERENCE APPROACH FOR MMIC DESIGN

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### ABSTRACT

The efficiency of the Finite-Difference method is improved by combining the full-wave analysis with a quasi-static approach: Those regions of a structure which require a spatial resolution far below the wavelength are described by a quasi-static analysis. As a consequence, the mesh size of the dynamic problem and hence the numerical efforts can be reduced significantly. The savings are particularly high for miniaturized geometries such as used in coplanar MMIC geometries.

### INTRODUCTION

The design of microwave and millimeter wave integrated circuits requires efficient and accurate CAD tools. Among them, the field-oriented simulation gains importance, which is a result of both the trend towards higher packaging density and the necessity to include housing effects. To this end, one would like to analyze the entire chip by a 3D approach. On the other hand, the smallest dimensions to be included are in the range of microns, such as the metallization thickness in coplanar MMICs. Given the simulation methods and computer facilities available so far, it is impossible to cover this extremely wide range in spatial resolution due to excessive numerical efforts. Therefore, the primary goal in developing field-oriented MMIC simulation methods is to improve efficiency.

This paper presents a new Finite-Difference frequency domain approach to solve this problem. It is tailored to the MMIC-typical situation and takes advantage of the fact that the finest resolution required is by orders of magnitude smaller than the

wavelength. Thus, a large part of the structure can be analyzed with good accuracy by using quasi-static assumptions. Therefore, we propose a hybrid formulation where the geometrical details are treated by a static approach using a fine mesh and the dynamic problem is solved only on a relatively coarse grid. This yields a considerable reduction in computational efforts.

### METHOD OF ANALYSIS

The hybrid FD scheme consists of the following two-step procedure (Fig.1 illustrates the different levels of discretization):

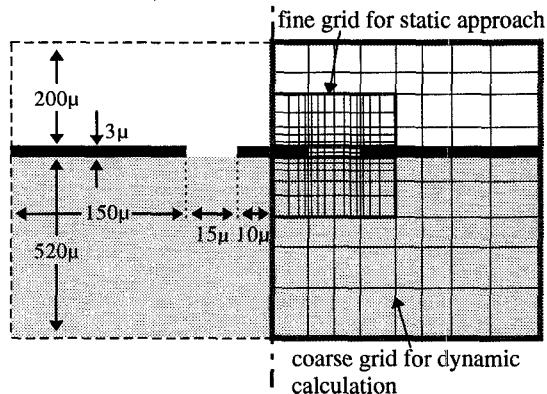


Fig.1: CPW Cross-section: the different levels of discretization for the static and the dynamic description

The structure (or only critical subregions) is analyzed by a static FD method with high resolution, i.e., a dense mesh. The numerical expense is much lower than for the corresponding full-wave solution. Additionally, in the lossless case, the static data do not change with frequency. Thereby, when varying the frequency, this part of the analysis needs to be calculated only once. The static results are incorporated into the dynamic FDFD analysis by means of weighting factors for the integrals over the elemen-

ary cells. Then the complete problem is solved on a coarse mesh.

Our approach is based on the integral formulation of the Finite-Difference frequency domain (FDFD) method [1], [2]. The Maxwellian equations are written in the form of integrals over the edges and surfaces of the elementary cells. The principal of our hybrid approach is that we use the static information (or any other a-priori knowledge of the field behavior) to improve the integral approximation.

In the conventional scheme, the integrals are calculated simply by multiplying the field value in the center with the respective cell length. This approximation yields the discretization error  $\delta_d$ , e.g. for the line integral of  $E_x$ :

$$\int_{x_m}^{x_m+\Delta x_m} E_x(x) \cdot dx = E_x(x_m + \Delta x_m / 2) \cdot \Delta x_m \cdot (1 + \delta_d) \quad (1)$$

Fig.2 illustrates the error due to the conventional FD-integral approximation.

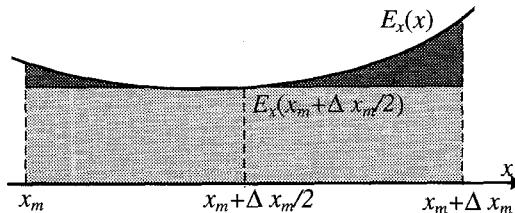


Fig.2: Line integral over elementary cell: conventional FD approach (grey area); dark: discretization error

It is clear that the resulting error in the integral approximation can be eliminated if the true mean value is applied in eqn.1 instead of the cell-center value  $E_x(x_m + \Delta x_m / 2)$ . In many cases however, the mean value can be determined with good accuracy by a static analysis. Therefore, a weighting factor  $\Lambda^{ex}$  is generated from the static field distribution  $E_{xs}(x)$

$$\Lambda_m^{ex} = \frac{\int_{x_m}^{x_m+\Delta x_m} E_{xs}(x) \cdot dx}{E_x(x_m + \Delta x_m / 2) \cdot \Delta x_m} = (1 + \delta_{ds}) \quad (2)$$

and its discretization error  $\delta_{ds}$  respectively. The improved description of the integral in eqn.1 then reads

$$\int_{x_m}^{x_m+\Delta x_m} E_x(x) \cdot dx = \Lambda_m^{ex} \cdot E_x(x_m + \Delta x_m / 2) \cdot \Delta x_m \cdot (1 + \delta_r) \quad (3)$$

with the residual error  $\delta_r$ :

$$\delta_r = \frac{\delta_d - \delta_{ds}}{1 + \delta_{ds}} \quad (4)$$

Due to the fact that the relative dynamic and static field distributions approach in critical subregions, such as metallic edges, the difference  $\delta_d - \delta_{ds}$  and hence the resulting error  $\delta_r$  remains small. In this way, weighting factors  $\Lambda$  for all line integrals and  $\Phi$  for all surface integrals are introduced in the dynamic FD approach. This leads to a substitution of the field values in the FD-algorithm:

$$E \rightarrow [\Lambda^e] \cdot E = \tilde{E} ; \quad B \rightarrow [\Phi^b] \cdot B = \tilde{B} \quad (5)$$

Now the electric field values are interpreted as line integrals, while the magnetic field values are interpreted as surface integrals. The local averages of the tensors of  $\epsilon$  and  $\mu$  are also substituted.

$$\begin{aligned} [\epsilon] &\rightarrow [\Phi^e] \cdot [\Lambda^e]^{-1} \cdot [\epsilon] = [\tilde{\epsilon}] ; \\ [\mu] &\rightarrow [\Lambda^b] \cdot [\Phi^b]^{-1} \cdot [\mu] = [\tilde{\mu}] \end{aligned} \quad (6)$$

Thus the discretization errors of a FD simulation using a coarse mesh are compensated by effective anisotropic material parameters. While the type of weighting factor formulation is related to [3,4] we now have an uniform two-grid-level FD approach with both static and dynamic data derived by the FD scheme. Furthermore, the static field information is utilized not only in the edge regions but over subsections or even the entire structure. This enables one, for instance, to discretize the slot of a MMIC coplanar waveguide with only one step in the dynamic solution without sacrificing accuracy. It should be pointed out that the inclusion of the weighting factors as described above preserves consistency of the FD approach. Therefore, mode orthogonality, energy conservation, and similar theorems are fulfilled for the discretized system within numerical accuracy, as in the conventional FD approach. This feature is important, for instance, with regard to S-parameter extraction.

## RESULTS

The new formulation was implemented into a FORTRAN code [4] to investigate efficiency and numerical properties. In order to evaluate the

improvement against the conventional approach the propagation characteristics for typical MMIC geometries are studied.

Fig.3 shows the relevant data for the propagation constant of the CPW structure of Fig.1: The new approach is compared to the conventional method (the percentage values refer to a conventional FD simulation with a mesh density five times higher). For both the conventional and the hybrid approach, the same coarse mesh is used for the dynamic solution. The hybrid method, however, employs a static approach on a grid refined by a factor of five. Fig.3 demonstrates that this improves accuracy considerably.

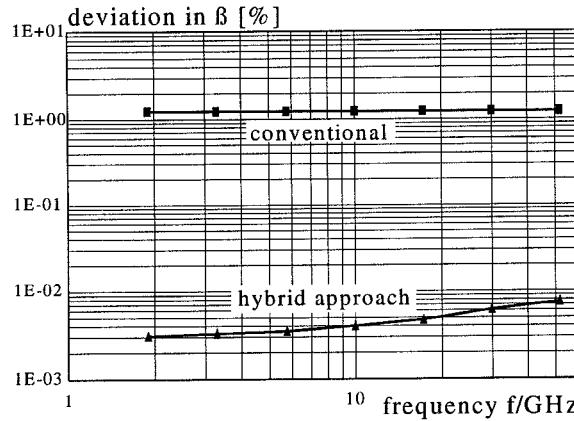


Fig.3: Percentage deviation in  $\beta$  against frequency for the CPW structure of Fig.1: solutions of conventional FD method and the new hybrid approach; the deviations refer to a full-wave analysis with a five times higher resolution.

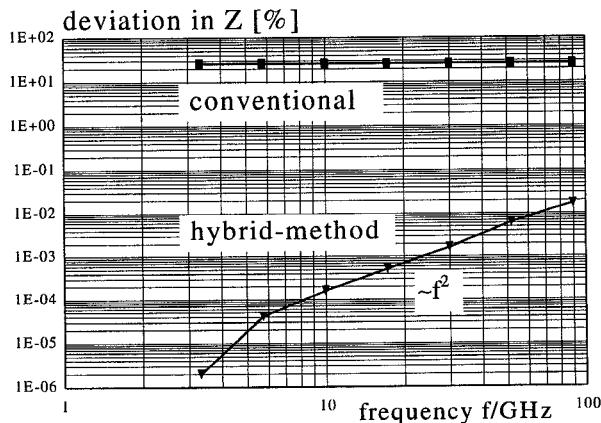


Fig.4: Data of Fig.3 for characteristic impedance Z

Fig.4. presents the corresponding data for the characteristic impedance Z. The improvement is even better

here. As can be seen the error of the hybrid method follows a  $f^2$  rule. This is clear since the discrepancies between static and dynamic fields increase with growing frequencies.

The advantages of the hybrid method are not restricted to CPW structures as Fig.5 demonstrates for the microstrip case.

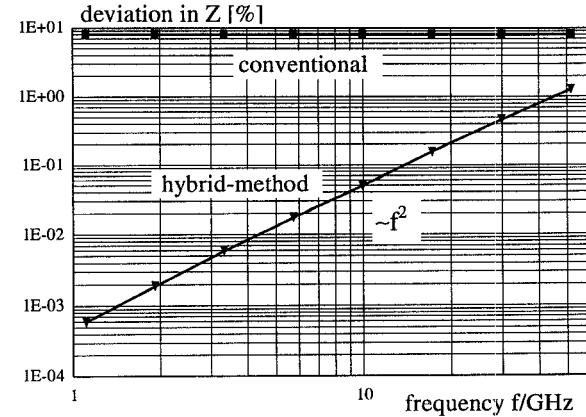


Fig.5: Percentage deviation in characteristic impedance Z against frequency for a microstrip structure: comparison between conventional FD method and hybrid approach. (for other data see Fig.4).

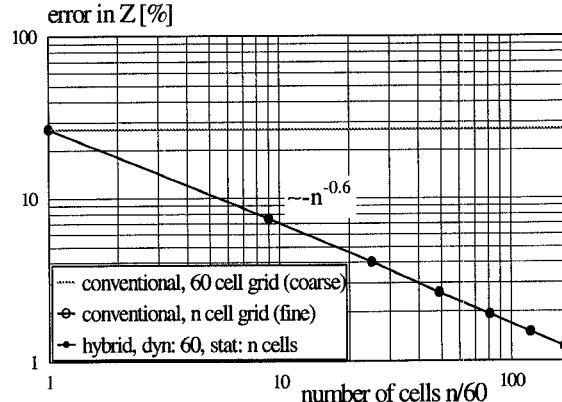


Fig.6: Convergence behaviour: percentage error in characteristic impedance Z against number of cells  $n$ ; hybrid approach with fixed dynamic grid (60 cells) and static mesh size varying ( $n$  cells) compared with conventional solutions for 60 cells and  $n$  cells. (CPW structure as in Fig.1).

Fig.6 provides information on convergence. The error in characteristic impedance Z is plotted against the number of cells in the CPW cross section. For the hybrid approach, a coarse dynamic mesh with  $n=60$  cells is applied while the static grid is varied with  $n$ . Regarding the conventional method, the solution for

the coarse mesh as well as that for the fine static grid are included. As can be seen, the hybrid solution with a fixed dynamic mesh converges to the correct value when refining the static grid. Furthermore the resulting error equals that of the full-wave solution on the fine grid although the latter one is computationally much more expensive.

Finally, Fig.7 illustrates the improvement in numerical efficiency compared with the conventional FDFD approach. Given the mesh size  $n$ , both approaches yield the same level of accuracy but the conventional method requires more than one order of magnitude more CPU time. This difference increases further when more than one frequency point is treated because the static calculation does not need to be repeated. Using a higher number of mesh cells the difference increases as well since the solution of a static problem is faster than that of its full-wave counterpart.

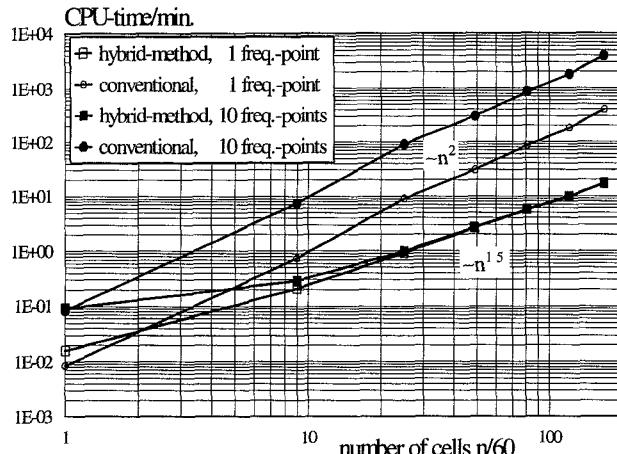


Fig.7: CPU time of the hybrid and the conventional method against number of cells (the data were generated on a DEC 3000/800 Alpha Workstation).

As can be seen, considerable savings in CPU time are obtained. This is true also for the storage which is related to the mesh size  $n$ . Because of sparsity the matrix size of both conventional and hybrid approach grows linearly with  $n$ . Due to the simplified mathematics in the static case, however, this type of analysis consumes only 30% of the corresponding full-wave problem. Furthermore, only the critical sub-regions of a structure need refinement. This further increases the efficiency of the hybrid approach.

## CONCLUSIONS

Summarizing the results the following conclusions with regard to field-oriented MMIC simulation can be drawn:

The hybrid static-dynamic FDFD formulation allows for local mesh refinement while preserving the essential features of the conventional Yee-scheme (consistency, energy conservation, mode orthogonality in waveguides). For typical MMIC CPW structures, the computational effort is reduced by more than one order of magnitude in CPU time and by 2/3 in storage. This represents one step further towards wafer-scale field-oriented MMIC design. Work is in progress now in order to extend the new approach to 3D analysis and the calculation of scattering coefficients.

## ACKNOWLEDGEMENTS

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## REFERENCES

- [1] K.S. Yee, "Numerical solution of initial boundary value problems involving Maxwell's equations in isotropic media," *IEEE Trans. Antenna Propagat.*, Vol. AP-14, pp. 302-307, May 1966.
- [2] T. Weiland, "On the numerical solution of Maxwellian eigenvalue problems in three dimensions," *Particle Accelerators*, Vol. 17, pp. 227-242, 1985.
- [3] D.B. Shorthouse and C.J. Railton, "The incorporation of static field solutions into the Finite Difference time domain algorithm," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-40, pp. 986-994, May 1992.
- [4] K. Beilenhoff and W. Heinrich, "Treatment of field singularities in the Finite-Difference approximation," *1993 International Microwa-ve Symposium Digest*, Vol. 2, pp. 979-982.
- [5] K. Beilenhoff, W. Heinrich, and H.L. Hartnagel, "Improved Finite-Difference formulation in the frequency domain for three-dimensional scattering problems," *IEEE Trans. Microwave Theory Tech.*, Vol. MTT-40, pp. 540-546, March 1992.